where $\dot{m}_t$ is the total flux ($\dot{\mu} = \dot{m}_t$ between x_n and x) The jet-damping force is now

$$R_y = -2\omega \dot{m}_t \left\{ \int_{x_0}^{x_n} \frac{x - x_0}{x_n - x_0} dx + (x - x_n) \right\}$$

$$= -2\omega \dot{m}_t [x - \tfrac{1}{2}(x_n + x_0)] \quad (9)$$

For the jet-damping moment a similar calculation yields

$$N = -\omega \dot{m}_t [x^2 - \tfrac{1}{3}(x_n^2 + x_n x_0 + x_0^2)] \quad (10)$$

If the result is compared with Eq (6) in the special case $x_n = x_e$, $x_0 = 0$, the value given by Eq (10) is different by a factor $\frac{2}{3}$, which is not negligible

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Vortex Flow in Arc Heaters

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Introduction

As a result of the increasing use of electric arc heaters, both to generate high-enthalpy streams for testing purposes and for space propulsion, various investigators (e g , Refs 1–4) have attempted to analyze the detailed mechanism of the energy exchange between the electric arc and the gas stream In such analyses the arc column geometry is postulated, and various flow regimes are considered separately Despite its simplifications, the Stine-Watson theory,[2] for example, appears to provide good results for the so-called constricted-arc configuration, but in it, as well as in most other theoretical analyses, the flow is considered to be one-dimensional However, in many arc-heater designs, vortex flow is used to induce arc rotation and thereby minimize electrode erosion The purpose of this note is to summarize some effects of vortex flow on the operation of Gerdien-type and constricted-arc configurations and, as a result, attempt to indicate the validity of applying irrotational energy exchange theories to vortex-flow or gas-stabilized arc heaters

Discussion

A typical Gerdien arc-heater geometry is shown in Fig 1a and the very similar constricted-arc configuration in Fig 1b Since much of the energy addition occurs in a constant-area cylindrical passage for these configurations, it appears reasonable, as noted in Refs 5 and 6, to approximate the heating process gas dynamically by Rayleigh flow

Rayleigh-type heat addition may accelerate subsonic flows to sonic velocity If, however, the amount of heat transferred to the gas is greater than that required to produce sonic velocity at the exit of the constant-area heating section, which would normally be the case in an arc heater, some adjustment of the entry conditions must occur Before striking an arc the cold gas is accelerated, nearly isentropically, from the arc chamber stagnation pressure and temperature to the nozzle exit static pressure and temperature The cold flow can already be sonic at the exit of the constant-area nozzle passage and nearly sonic at the entrance Consequently, the addition of heat to the gas, which begins when the arc is struck, must cause some readjustment of the flow conditions entering the constant-area passage As pointed out by Shapiro,[7] this adjustment is brought about by a series of transient pressure waves which propagate in both directions along the channel, changing either or both the mass flow and pressure level until the velocity or Mach number at the entrance of the constant-area section decreases to allow the required amount of heating to occur with sonic velocity at the exit

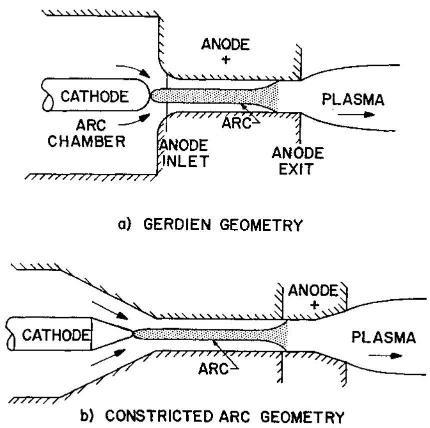

Fig 1 Arc-heater configuration

From a Rayleigh analysis one would conclude that arc heating rather than gasdynamic expansion is primarily responsible for the acceleration of the plasma to $M = 1\ 0$ at the exit of the constant-area section This effect is clearly demonstrated by the anode pressure distributions shown in Fig 2, which were obtained with a Gerdien configuration (see Ref 6 for details) Without an electric arc, the cold flow enters the anode passage at a Mach number of about 0 7 and is accelerated to sonic velocity at the exit by frictional effects The pressure distribution demonstrates that, when the arc is initiated, the flow readjusts itself to a very low anode entry Mach number ($M \ll 0\ 1$) and is accelerated rather uniformly by the heat addition When the gas is introduced tangentially into the arc heater to produce a vortex flow, the resulting anode pressure distributions follow the same pattern Since the heat-addition process greatly decreases the axial velocity component at the entrance of the anode passage, the velocity in the arc-heater stilling chamber is greatly decreased Consequently, it seems reasonable that the tangential velocity component for the vortex-flow case would also be significantly decreased because of the longer "dwell time" and increased viscous dissipation in the stilling chamber Also, when the mass flow decreases as a result of heat addition, the tangential injection velocity decreases Thus, it might be expected that the arc heat addition through either of the mechanisms would tend to weaken the vortex or any other flow nonuniformities arising in the arc-heater stilling chamber

Some impact pressure surveys, which illustrate this point, have been made in free jets from arc heaters of the geometry shown in Fig 1a Pressure profiles taken at two nozzle diameters from the exit plane of a sonic jet are shown in Fig 3 for vortex and nonvortex flows (see Ref 6 for details) For the vortex case, argon was injected tangentially into the arc chamber at a velocity of about 100 fps From angular momentum considerations a tangential velocity of about 500 fps would be expected at the wall of the anode passage for

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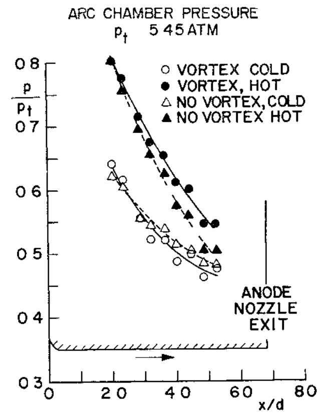

Fig 2 Anode pressure distributions

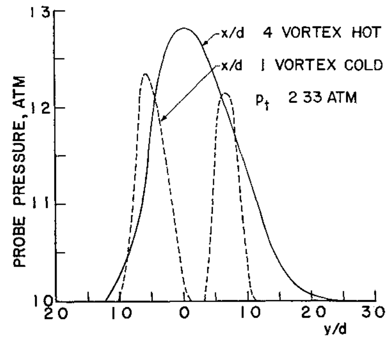

Fig 4 Impact pressure profiles with and without a strong vortex [8]

cold flow The typical profiles in Fig 3 show that the impact pressures for nonvortex, cold flow are relatively constant over the center portion of the jet, whereas a large pressure void exists in the center of the jet for cold vortex flow, as expected However, when the arc is struck and operated at a power level of about 11 5 kw, the effects of the vortex disappear, and the profile shows no evidence of rotational flow Other measurements have been made which demonstrated that other cold flow irregularities (in these experiments introduced by an attachment nut on the end of the cathode) were smoothed out by the arc heat-addition process

For the experiments just described, the tangential injection velocities were low, and thus the vortex was rather weak McBrayer[8] has made similar measurements (but with much higher injection velocities) to determine if much stronger vortices would persist through the minimum area passage Some results of his tests where the injection velocity was about 800 fps are shown in Fig 4 The vortex cold flow profile, made at one nozzle diameter downstream of the nozzle exit plane, demonstrated the effect of a much stronger vortex with subatmospheric pressures at the center of the jet With vortex flow the jet expanded very rapidly because of centrifugal effects such that at $x/d = 4$ there was no measurable dynamic pressure defining the jet boundary over the range traversed by the probe However, when the arc was established at a power level of 8 3 kw, the influence of the vortex was again eliminated, and the pressure profile, given by the solid line in Fig 4, was measured at $x/d = 4$ where no measurable profile existed for cold flow Thus, the experimental evidence supports the analysis and indicates that gasdynamic rotation is effectively damped by heat addition for Gerdien and constricted-arc configurations Moreover, the performance of these types of arc heaters would not be significantly different for vortex and nonvortex flows except at very low power levels where the heat addition is small Experimentally measured plasma enthalpy levels and arc-heater energy-conversion efficiencies given in Ref 6 show little difference between vortex and nonvortex flow Thus, it may be concluded that irrotational theories have some application to vortex-flow arc heaters of the Gerdien or constricted-arc configuration

One additional point might be made with respect to the heat-addition process For conventional Rayleigh flow there is a stagnation pressure decrease attributable to heat addition which can be as large as 23% of initial stagnation pressure If the plasma can be treated as a perfect gas and frictional losses are small, this pressure decrease can be easily predicted A comparison of the cold flow and plasma profiles for the nonvortex case in Fig 3 illustrates this loss This stagnation pressure loss should be taken into account in calculations such as for the plasma temperature from the continuity equation applied at the sonic point In Refs 5 and 6, the decrease of stagnation pressure is discussed in more detail, and procedures are outlined to incorporate this effect in arc-heater calculations Data presented in Ref 6 also illustrate that, as the input power (gas heating) increases, the stagnation pressure loss increases, rapidly at first, but then quite slowly as the Mach number at the entrance of the constant-area passage approaches zero

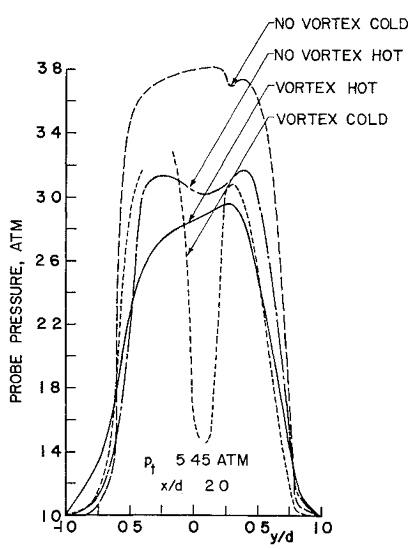

Fig 3 Impact pressure profiles with and without vortex flow [6]

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